N16/5/MATSD/SP2/ENG/TZ0/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 2

Friday 11 November 2016 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

7 pages

8816 – 7402

Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 17]

In the month before their IB Diploma examinations, eight male students recorded the number of hours they spent on social media.

For each student, the number of hours spent on social media (x) and the number of IB Diploma points obtained (y) are shown in the following table.

Hours on social media (x)	6	15	26	12	13	40	33	23
IB Diploma points (y)	43	33	27	36	39	17	20	33

(a) On graph paper, draw a scatter diagram for these data. Use a scale of $2\,\text{cm}$ to represent 5 hours on the x-axis and $2\,\text{cm}$ to represent 10 points on the y-axis. [4]

(b) Use your graphic display calculator to find

(i) $\bar{x}$, the mean number of hours spent on social media;

(ii) $\bar{y}$, the mean number of IB Diploma points. [2]

(c) Plot the point $(\bar{x}, \bar{y})$ on your scatter diagram and label this point M. [2]

(d) Write down the value of r, the Pearson's product–moment correlation coefficient, for these data. [2]

(e) Write down the equation of the regression line y on x for these eight male students. [2]

(f) Draw the regression line, from part (e), on your scatter diagram. [2]

Ten female students also recorded the number of hours they spent on social media in the month before their IB Diploma examinations. Each of these female students spent between 3 and 30 hours on social media.

The equation of the regression line y on x for these ten female students is

$$y = -\frac{2}{3}x + \frac{125}{3}.$$

An eleventh girl spent 34 hours on social media in the month before her IB Diploma examinations.

(g) Use the given equation of the regression line to estimate the number of IB Diploma points that this girl obtained. [2]

(h) Write down a reason why this estimate is not reliable. [1]

2. [Maximum mark: 12]

A group of 66 people went on holiday to Hawaii. During their stay, three trips were arranged: a boat trip (B), a coach trip (C) and a helicopter trip (H).

From this group of people:

3 went on all three trips;
16 went on the coach trip **only**;
13 went on the boat trip **only**;
5 went on the helicopter trip **only**;
x went on the coach trip and the helicopter trip **but not** the boat trip;
$2x$ went on the boat trip and the helicopter trip **but not** the coach trip;
$4x$ went on the boat trip and the coach trip **but not** the helicopter trip;
8 did not go on any of the trips.

(a) Draw a Venn diagram to represent the given information, using sets labelled B, C and H. [5]

(b) Show that $x = 3$. [2]

(c) Write down the value of $n(B \cap C)$. [1]

One person in the group is selected at random.

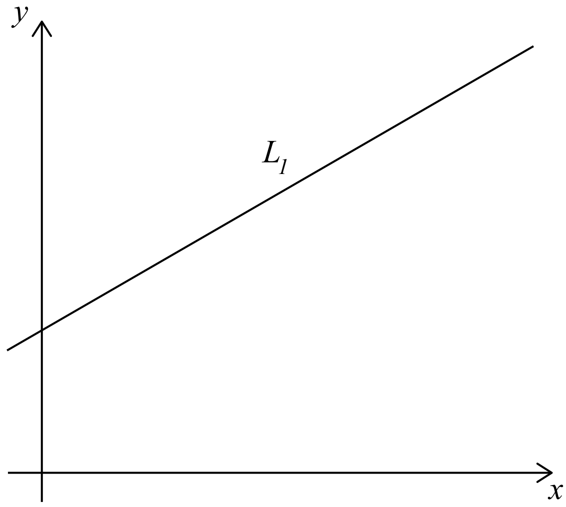
(d) Find the probability that this person

(i) went on at most one trip;

(ii) went on the coach trip, given that this person also went on both the helicopter trip and the boat trip. [4]

3. [Maximum mark: 17]

The line L_1 has equation $2y - x - 7 = 0$ and is shown on the diagram.



The point A has coordinates $(1, 4)$.

(a) Show that A lies on L_1. [2]

The point C has coordinates $(5, 12)$. M is the midpoint of AC.

(b) Find the coordinates of M. [2]

(c) Find the length of AC. [2]

The straight line, L_2, is perpendicular to AC and passes through M.

(d) Show that the equation of L_2 is $2y + x - 19 = 0$. [5]

The point D is the intersection of L_1 and L_2.

(e) Find the coordinates of D. [2]

The length of MD is $\frac{\sqrt{45}}{2}$.

(f) Write down the length of MD correct to five significant figures. [1]

The point B is such that ABCD is a rhombus.

(g) Find the area of ABCD. [3]

4. [Maximum mark: 11]

A manufacturer produces 1500 boxes of breakfast cereal every day.

The weights of these boxes are normally distributed with a mean of 502 grams and a standard deviation of 2 grams.

(a) Draw a diagram that shows this information. [2]

All boxes of cereal with a weight between 497.5 grams and 505 grams are sold.
The manufacturer's income from the sale of each box of cereal is $\$2.00$.

(b) (i) Find the probability that a box of cereal, chosen at random, is sold.

(ii) Calculate the manufacturer's expected daily income from these sales. [4]

The manufacturer recycles any box of cereal with a weight **not** between 497.5 grams and 505 grams. The manufacturer's recycling cost is $\$0.16$ per box.

(c) Calculate the manufacturer's expected daily recycling cost. [2]

A **different** manufacturer produces boxes of cereal with weights that are normally distributed with a mean of 350 grams and a standard deviation of 1.8 grams.

This manufacturer sells all boxes of cereal that are above a minimum weight, w.

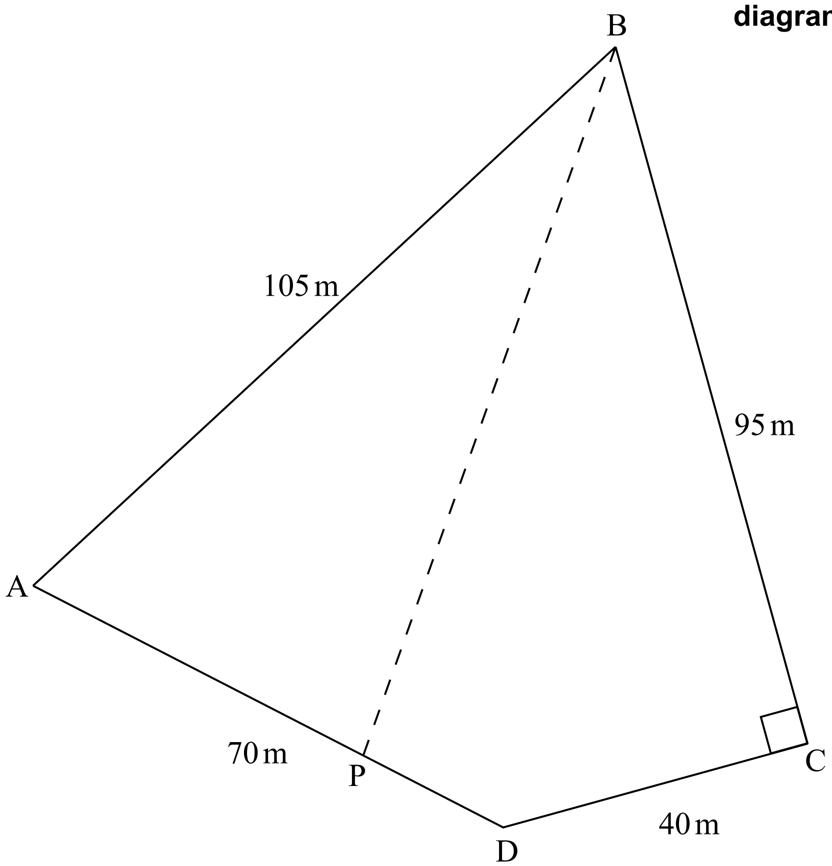
They sell 97% of the cereal boxes produced.

(d) Calculate the value of w. [3]

5. [Maximum mark: 16]

A farmer owns a plot of land in the shape of a quadrilateral ABCD.
$\text{AB} = 105\text{ m}$, $\text{BC} = 95\text{ m}$, $\text{CD} = 40\text{ m}$, $\text{DA} = 70\text{ m}$ and angle $\text{DCB} = 90^\circ$.

diagram not to scale



The farmer wants to divide the land into two equal areas. He builds a fence in a straight line from point B to point P on AD, so that the area of PAB is equal to the area of PBCD.

Calculate

(a) the length of BD; [2]

(b) the size of angle DAB; [3]

(c) the area of triangle ABD; [3]

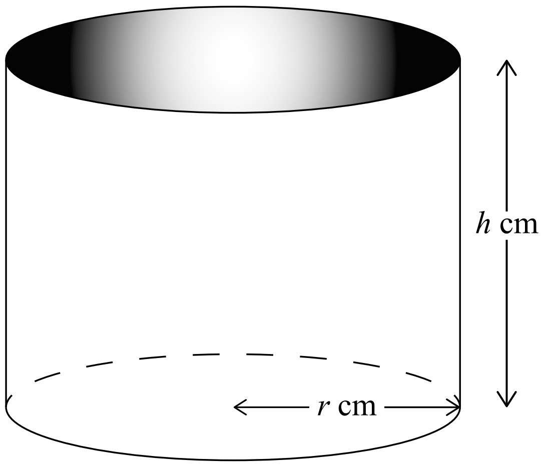
(d) the area of quadrilateral ABCD; [2]

(e) the length of AP; [3]

(f) the length of the fence, BP. [3]

6. [Maximum mark: 17]

A water container is made in the shape of a cylinder with internal height h cm and internal base radius r cm.



The water container has no top. The inner surfaces of the container are to be coated with a water-resistant material.

(a) Write down a formula for A, the surface area to be coated. [2]

The volume of the water container is $0.5\,\text{m}^3$.

(b) Express this volume in cm^3. [1]

(c) Write down, in terms of r and h, an equation for the volume of this water container. [1]

(d) Show that $A = \pi r^2 + \dfrac{1000\,000}{r}$. [2]

The water container is designed so that the area to be coated is minimized.

(e) Find $\dfrac{\mathrm{d}A}{\mathrm{d}r}$. [3]

(f) Using your answer to part (e), find the value of r which minimizes A. [3]

(g) Find the value of this minimum area. [2]

One can of water-resistant material coats a surface area of $2000\,\text{cm}^2$.

(h) Find the least number of cans of water-resistant material that will coat the area in part (g). [3]